



Impact of Variable Speed Limits and Shoulder Lane Openings on Traffic Improvement near Merging Zones under Free Flow Condition

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Article info	Abstract
Original: Revised: Accepted: Published online:	The distribution of traffic flow in the individual lanes of multilane motorways is an important investigation task in traffic engineering because the lane flow-distribution affects directly the capacity of the motorway section under investigation. This work is a data analysis which shows the impact of variable speed limits (VSLs) and dynamic lane (shoulder lane) on traffic improvement by homogenization flow through different lanes of a highway. Moreover, it indicates the main characteristics of lane flow distribution (LFD) on all lanes of traffic at the downstream of an on-ramp on a highway. It is concluded that the VSLs can homogenize traffic flow on all lanes of a carriageway by increasing the flow on underused lanes (i.e. slow lane) and decreasing the flow on overloaded lanes (i.e. median lane). Furthermore, it is shown that dynamic lane (i.e. shoulder lane) management can improve the traffic situation by supporting a portion of the flow (about more than 10%) during the peak periods.
Key Words: Lane flow distribution Variable speed limit Traffic homogenization Highway traffic stream characteristics	

I. Introduction

Traffic is a non-homogenous phenomenon; the sizes of vehicles are different, the spacing between vehicles is not uniform and traffic flow is not equally distributed along the lanes. These non-homogeneities are one of the causes that the capacity of the highway is not fully exploited.

The distribution of traffic flow in the carriageway of a motorway is an important investigation task in traffic engineering, because the lane flow-distribution affects directly the capacity of the motorway section under investigation [5]. Several studies have been carried out describing the lane flow distribution (LFD). However, many of these studies focus on equilibrium without the influence of shoulder lane.

This paper specifies the actual effects of the variable speed limits (VSLs) in terms of empirical lane flow distribution at downstream of an on-ramp and specifies also, the influence of dynamic lane (shoulder lane) on lane flow distribution. Eventually, it deals with the effect of different VSLs on LFD% when the dynamic lane is open.

This paper firstly presents a literature review, section (II), on lane flow distribution and variable speed limits. Secondly, section (III) shows the set-up of the research including data collection (site, type of data) and the analyses which are carried out. Thirdly, the results lane flow distributions under the influence of variable speed limits and shoulder lane are presented in section (IV). Finally, the conclusion is in section (V) which

interprets these results and discusses the possible implication of VSLs and shoulder lane on traffic flow patterns.

II. LITERATURE REVIEW ON LANE FLOW DISTRIBUTION AND VARIABLE SPEED LIMIT (VSL)

It has been proven that the amounts of traffic flows of highway lanes are not homogenous. Therefore, the distribution of flow along the lanes varies depending on the amount of traffic flow. In the terms of better understanding the behavior of traffic on highways, further studies on this phenomenon (LFD) should be carried out.

Variable speed limits (VSLs) changes based on road traffic, and weather conditions. It slows down traffic ahead of congestion or bad weather to smooth flow. It diminishes stop-and-go conditions and reduces crashes. Consequently, this technique alerts drivers in real time to change their speed in function of the situation of the downstream on the road. It has proven that more consistent speeds improve safety by preventing rear-end and lane-changing collisions due to sudden stops [1]. During last 10 years, researches have been done using VSLs to improve the homogenization of traffic flow on a highway. LFD can be homogenized for different quantity of traffic flow using VSL: In 2010, Lee [2] found the LFD under different traffic conditions on basic segments of freeways. He proposed a density as a measurement to find accurate lane flow distributions. He also found that in uncongested condition, LFD of median lane had increased continuously and all flow ratios become close to each other when congestion was increased. In 2010, Duret [3] observed the LFD for 3 lanes highway (i.e. motorway) under free traffic flow condition. It was shown that, for a range of flow, the LFD of the slow and center lanes decreased linearly. On the contrary, the LFD of median lane increased linearly. This phenomenon was observed when the VSL was either activated or not activated. Knoop et al [4] discussed the change in lane distribution when VSL is implemented. They also observed the influence of an on-ramp on lane flow distribution. It was found that VSL increases the use of the outside lane (slow lane) near capacity. In addition to the fact that, the main point argued in this paper was the importance of effects of a variable speed limit on the lane flow distribution. Wu [5] studied the flow-distribution in the individual lanes and extended to carriageways with more than three lanes. The obtained model is then calibrated. The proposed model can cope with free flow and congested traffic conditions as well. Nissan et al [6] found that when the VSL is advisory, the VSL did not have any significant impact on traffic conditions, both immediately after its implementation and several months later. Li et al [7] proposed a suitable solution for mitigating congestion calling for effective management of existing facilities using an integrated traffic control system (ramp metering, VSLs and hard-shoulder running (HSR)), it is proven that HSR is an effective approach in reducing total delay by providing an extra physical lane to motorways. However, safety issues should be taken into account because using shoulder lane removes the buffer intended for incident management. In 2012, Duret et al [9] studied the lane flow distribution at a freeway. In this research, authors show that a driving ban for trucks and variable speed limits profoundly impact the lane flow distribution and increase utilization of the slow lane by reducing the speed difference between the slow and the median lane. In 2017, Pompigna et al [10] Proposed a nonlinear regression model of the distribution relationships for each lane, for both flow and density ratios, as a function of the total flow. From this study, the nonhomogeneous use of freeway lanes in Italy is qualified and quantified through the trends found in these ratios. The research is useful in terms of providing an interpretation of driver behavior, and is also a reference for calibrating traffic models. In 2013, Samoili et al [11] a short-term prediction models to provide a timely and more effective temporary hard shoulder activation are developed. The model is calibrated using real data collected with seven radar sensors located every 500m.

III. TRAFFIC DATA

In this paper, all the used data has been collected on the M42 motorway in the United Kingdom. Data collected for period between (5 AM-10 PM) in October and November. M42 is an important motorway in UK; it is a part of ring road (peripheral) of Birmingham as shown in (Figure 1). This motorway is composed

of three lanes permanent with one lane shoulder which is closed/open depending on the amount of traffic flow as it is demonstrated in (Figure 2). In M42, traffic flow often exceeds 140,000 vehicles per day during the working days. In this context, M42 is managed by an efficient system called Active Traffic Management (ATM). Hence, in order to collect the data used in this paper it has been acquired benefit from this system of traffic management.

In the rest of this paper, the indices (V0, V1, V2, and V3) are shoulder, slow lane, center lane and median lane, respectively. It should be noted that, the data is collected on a motorway in UK where English people circulate left as shown in (Figure 2).

As it was mentioned previously, the ATM system is based on the dynamic use of the infrastructure. This means when the traffic flow is high, the variable message sign, which controls speed on each lane, will be on over the shoulder and it could be used as a circulating lane. However, the maximum speed limit is 70 mph, when the ATM system is active; the speed limit is reduced to either 50 mph or 60 mph. Note that more than 30 VMS (variable message sign) are installed each 500 meters through the motorway to give direct information to the road users.

Data have been collected using detectors installed in the road structure each 100 m. The first detector is installed just at the end of the insertion lane after an on-ramp. In our study, we used 10 detectors spaced each 100 meters as indicated in both (Figures 1 and 2). The data recorded includes the passing time, the speed, the length of each vehicle, the position of the vehicle on the motorway laterally (i.e. on which lane it is) and longitudinally (i.e. how far it is from the benchmark point).

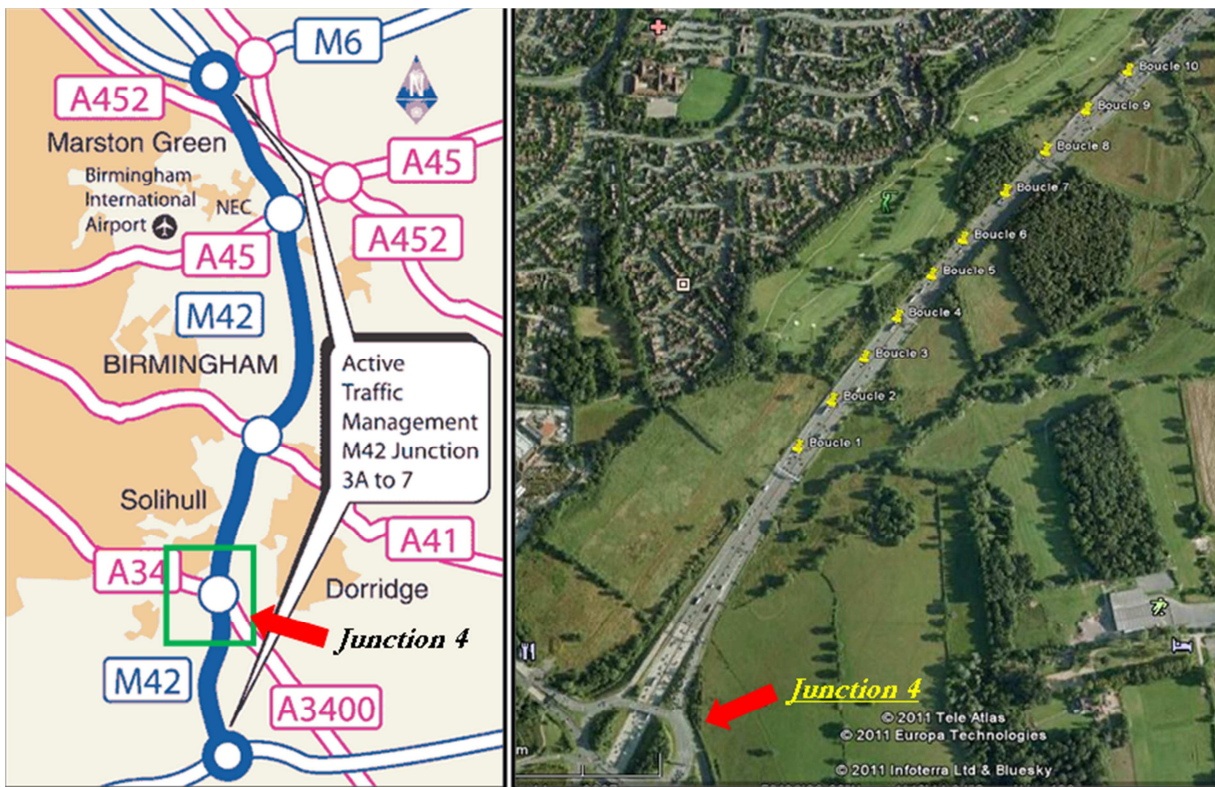


Figure 1 : M42 motorway near junction 4 [8]

IV. PRESENTATION OF TRAFFIC SITUATION ON THE MOTORWAY

In this section, traffic situation is illustrated on the motorway during the days of collecting data (October and November 2008), which has been collected by British Idris Diamond Consulting company. As an example; in (Figure 3), traffic flow [veh/h], speed [km/h], collected at detector No.5, is shown. At that day, the weather condition was ideal (i.e. the roads were dry) and the shoulder is open to circulation. Then, the diagram of fundamentals has been shown including (density-flow, flow-speed, and density-speed), as shown

in **(Figure 4)**. In order to eliminate the effect of heavy vehicles, we only analyze the data from 5:00 AM to 10 PM.

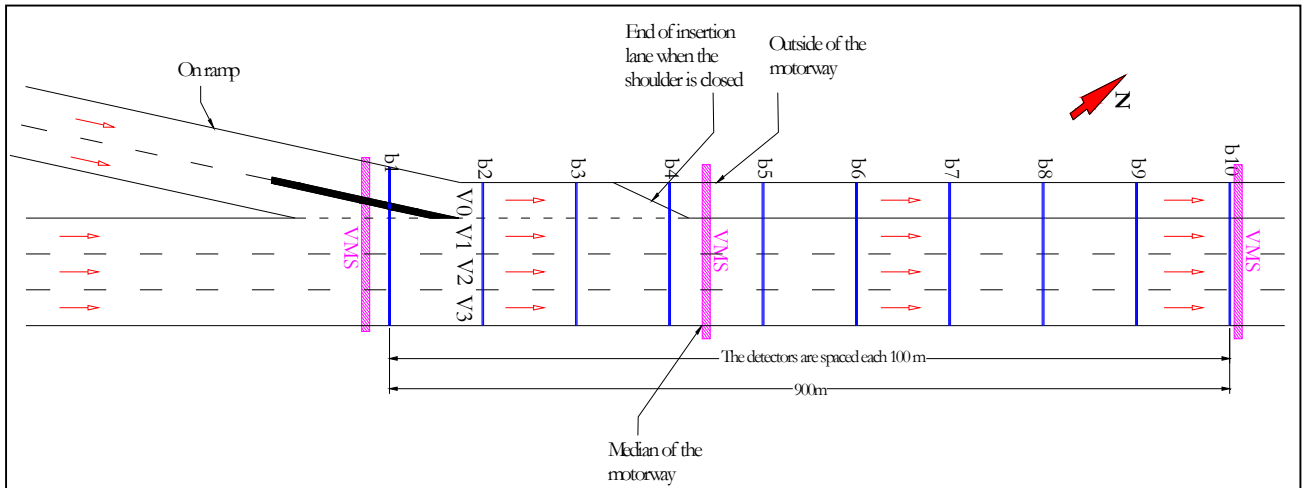


Figure 2: M42 motorway near junction

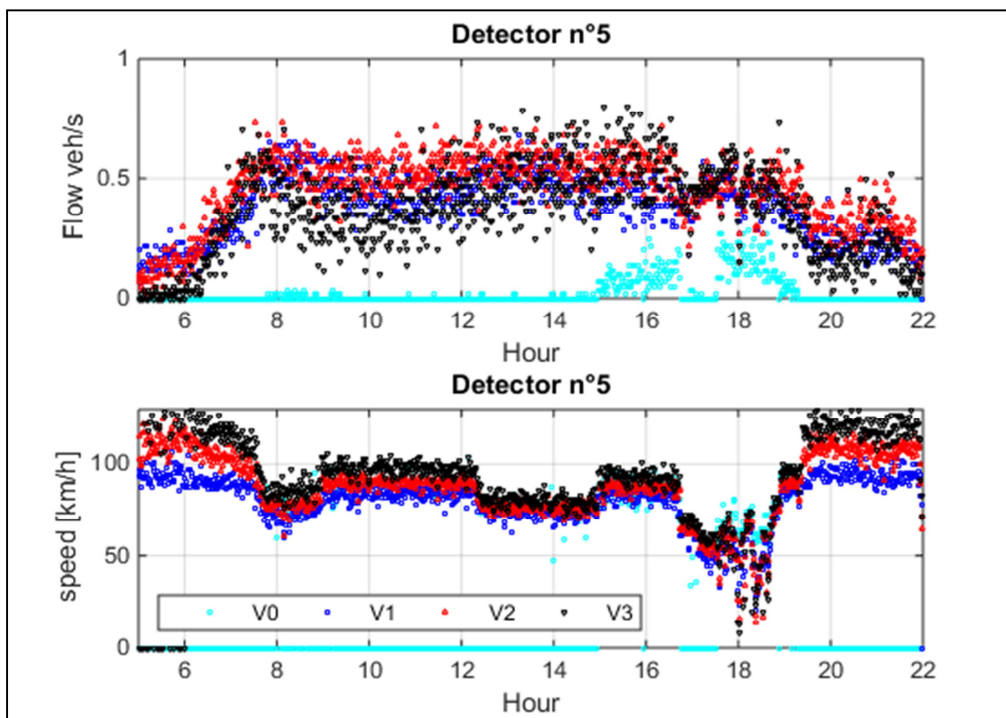


Figure 3: Traffic flow [veh/s] (Top) and mean speed [km/h] (Bottom) at detector no. 5. Cyan represents traffic flow at lane V0; Blue represents traffic flow at lane V1; Red represents traffic flow at lane V2; Black represents traffic flow at lane V3.

From Figure 3, It can be noted that:

1. Congestion appears from 4 PM to 8 PM and the speed value is then lower than 20 km/h.
2. The shoulder is open when congestion appears.
3. The VSL affect the motorway speed, for example, from 9 AM to 0:30 PM, the speed of lane V3 is limited to about 100 km/h while from 0:30 PM to 3 PM the speed falls to 80 km/h.
4. The lanes have different speeds.

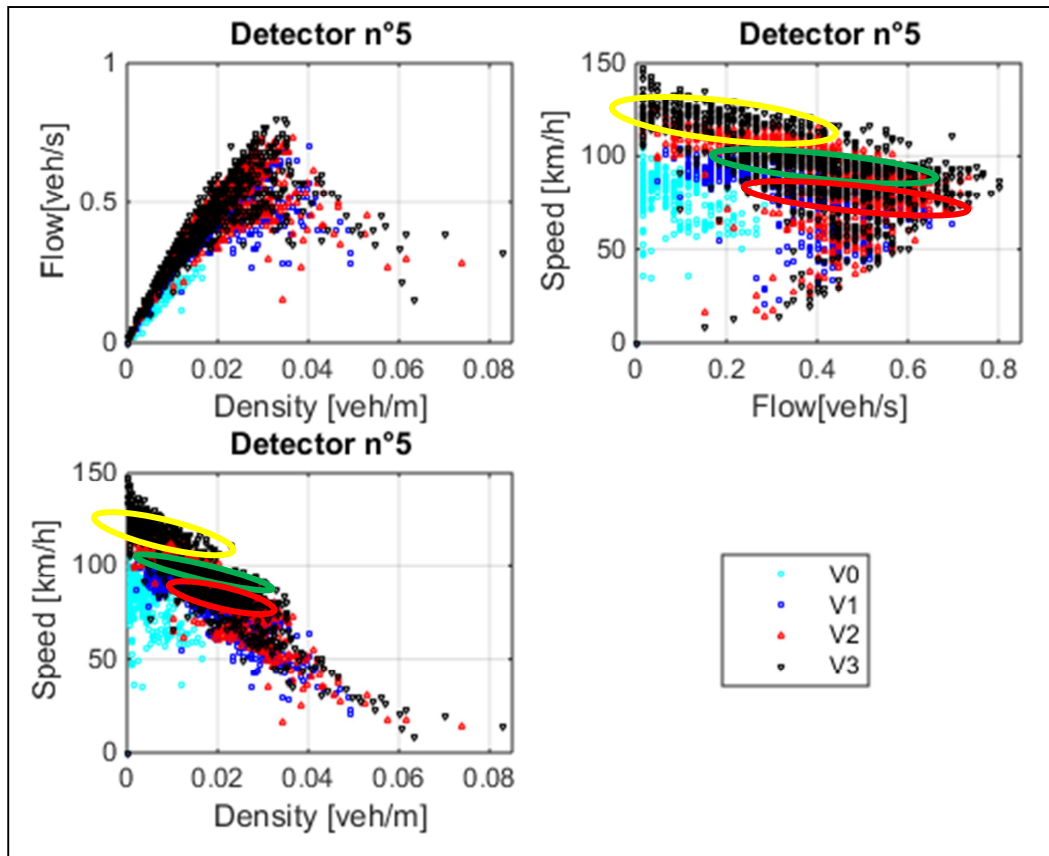


Figure 4: Fundamental diagram (flow-density (F-D), speed-flow (S-F) and speed-density(S-D)) at detector no. 5.

From Figure 4, it is shown that:

- I. In fluid zone of fundamental diagram, empirical data (i.e. collected data) could be presented by a straight line (see F-D relation).
- II. The free flow speed is not the same on all lanes. For example, the free flow speed on lane V0 is less than the free flow speed on other lanes (see F-S relation).
- III. It is feasible to distinguish between different speeds ranges of VSL (50 mph, 60 mph and 70 mph). For instance (see S-F and S-D relations), the speed range at lane V3 is 70 mph (ellipse yellow), 60 mph (ellipse green) and 50 mph (ellipse blue).

V. DATA FILTERING

The collected data contain fluid and congested traffic situation, which was collected in different condition of meteorology when the motorway was wet (i.e. rainy time) and dry (i.e. dry weather time). The shoulder was open in certain times when the congestion appears therefore, the data filtering was necessary. In this case, the data were filtered in order to select data when the traffic situation is fluid and the motorway is dry. Accordingly, in both cases, they were different when the shoulder is open and when it is closed, see (Figure 5). As a consequence, the data aggregated in period of one minute. This time space is quite short to present a steady state demand and quite long to estimate average mean speed on each circulating lane.

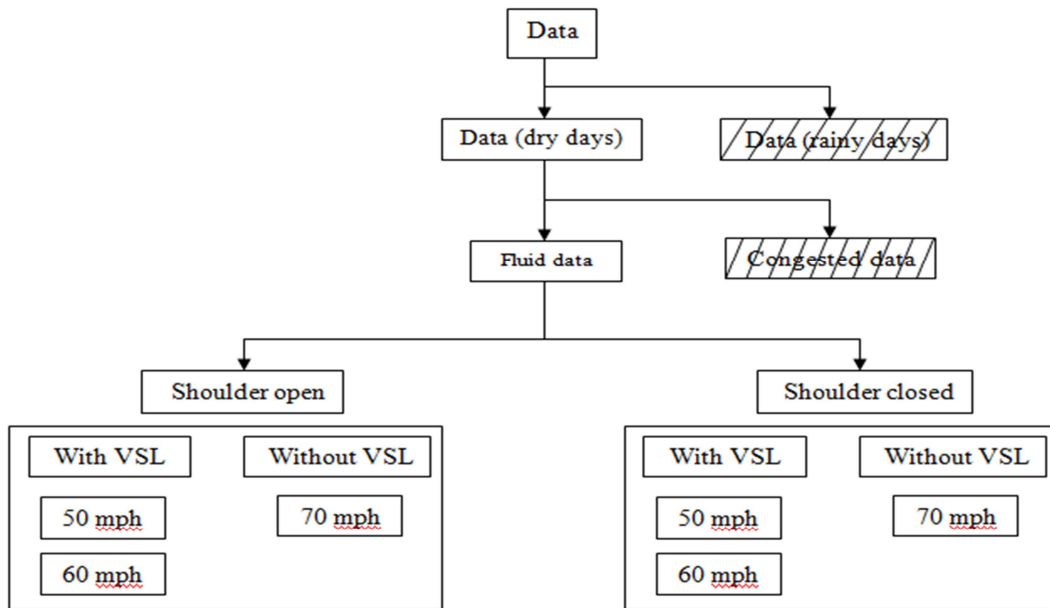


Figure 5: Data filtering of traffic data samples

Then, on the S-F diagram, the zones of fluid and congested are identified when the shoulder is open and/or closed. The limits which separate fluid zone from congested zone are found on each circulating lane, see (Table 1). Admittedly, the selection of the time periods when the shoulder lane is open, is based on the maximum value of the traffic flow on this lane (V0), see (Table 2).

Table 1: Limits which separate free flow zone from congested zone on each circulating lane from detector no.1 to detector no. 10; when the shoulder is open or closed.

Limits between free flow zone and congested zone on all detectors [km/h]					
		with / without shoulder	with / without shoulder	with / without shoulder	with / without shoulder
Lane \ VSL		V0	V1	V2	V3
50mph		>69	>69	>72	>77
60mph		>75	>75	>81	>88
70mph		>84	>84	>94	>109

Table 2: Traffic flow limit [veh/s] on shoulder lane when it is open or closed from detector 6 to detector 10

		Flow [veh/s] from detector 6 to detector 10	
		Open	Closed
Shoulder \ VSL			
50mph			
60mph		>0	0
70mph			

VI. DATA REPRESENTATION UNDER DIFFERENT INTERVAL OF FLOW

Notably, after filtering the data, it can be identified the periods when the traffic is free flow and congested. As a result, it is noticeable that in the congested period of traffic, there is no sufficient data to be analyzed in both open and closed shoulder cases. For this reason, the data is only analyzed when the traffic is free flow. In (Figure 6-a), the data samples have been illustrated in terms of average flow [veh/s] at detector No.1 when the traffic is fluid and the lane V0 is closed for VSL 50 mph, 60 mph and 70 mph, respectively. In this figure, one can note that:

- Data samples of VSL 50 mph and 60 mph are few between flow 0 and 1 [veh/s] and between flow 1.8 and 2.1 [veh/s]. On the contrary, data samples are sufficient between flow 1.0 and 1.7 [veh/s].
- On the other hand, data samples for VSL 70 mph are few between flow 1.4 and 2.1 [veh/s] but sufficient data samples are available between flow 0.1 and 1.3 [veh/s].

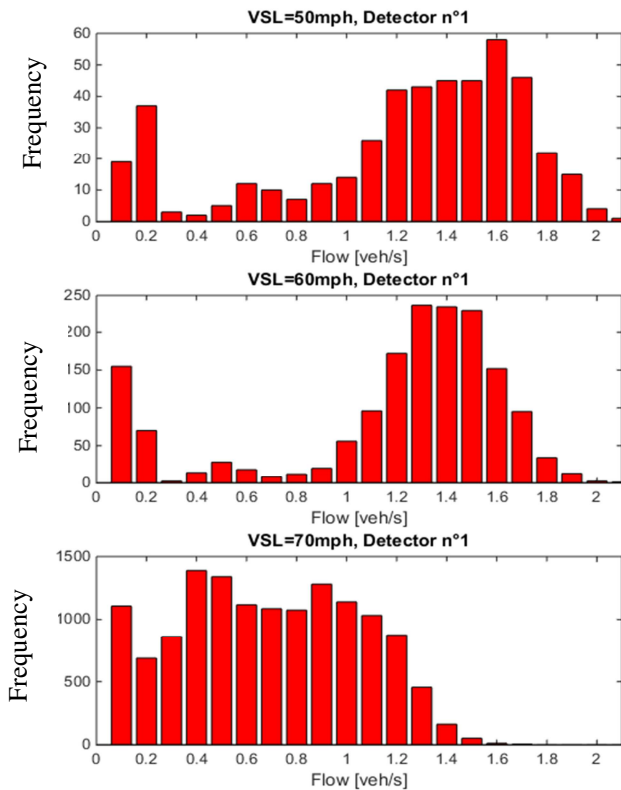


Figure 6-a

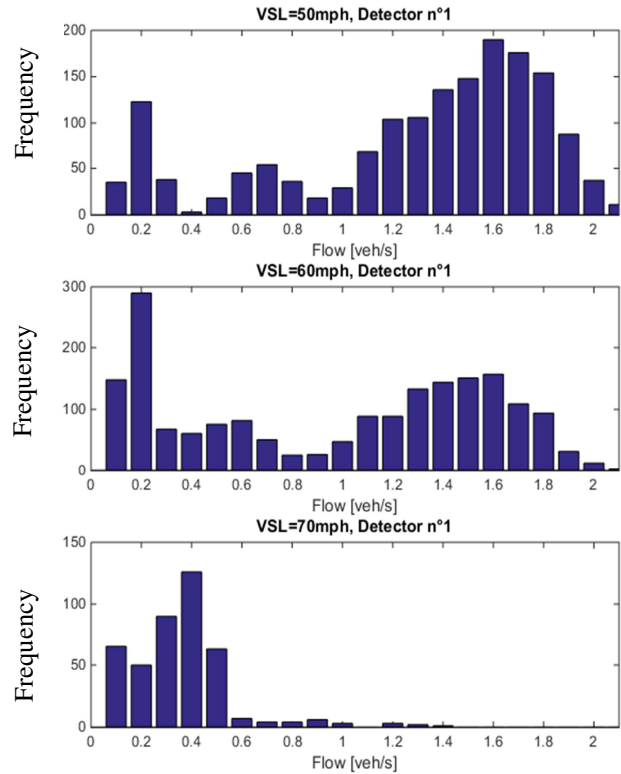


Figure 6-b

Figure 6: Data samples of 26 days (dry days) when traffic flow is free; the V0 is closed (left); the V0 is open (right) In (Figure 6-b), the data samples, in function of average flow [veh/s] at detector No.1, when the traffic is fluid and the lane V0 is open, are shown for VSL 50 mph, 60 mph and 70 mph, respectively.

- Firstly, the data samples of VSL with 50 mph are sufficient between flow 1.1 and 1.9 [veh/s]. Contrary, data samples are not quite enough between flow 0 and 1 [veh/s] and flow 2 and 2.1 [veh/s].
- Secondly, the data samples of VSL with 60 mph are sufficient between flow 0 and 0.7[veh/s] and flow 1 and 1.8 [veh/s]. On the flip side, data samples are not enough between flow 0.7 and 1[veh/s] and flow 1.8 and 2.1 [veh/s].
- Finally, the data samples of VSL with 70 mph are few between flow 0.5 and 2.1 [veh/s]. Conversely, there are enough data samples between flow 0.1 and 0.5 [veh/s].

VII. EFFECT OF TRAFFIC CONDITION ON LFD

- *Principles and characteristics of LFD:* It is known that the distribution of flow on the lanes varies depending on the infrastructure geometry for example, number of lanes and location (near to an on ramp or an off ramp). Here, the distribution of flow through lanes on a motorway section just behind an on-ramp as it is shown.
- *Lane flow distribution based on traffic flow:* The percentage of lane flow at each section can be calculated using the following equation:

$$\%LFD \text{ of } V_i = \frac{\text{flow of lane } V_i \text{ at time } t}{\sum \text{flow of all lanes at time } t}$$

Where $i = 0, 1, 2, \text{ and } 3$

LFD% = percentage of lane flow distribution

Figure 7 shows the percentage of flow per lane versus average flow [veh/s] on Friday 14 November at detector number 1. However, it is noticeable that when the total flow is low (about 0.5 [veh/s]), 10% of vehicles circulate on lane V0 (coming from the on ramp), 28% of vehicles circulate on lane V1, 45% of vehicles circulate on lane V2 and 17% which remain, circulate on lane V3.

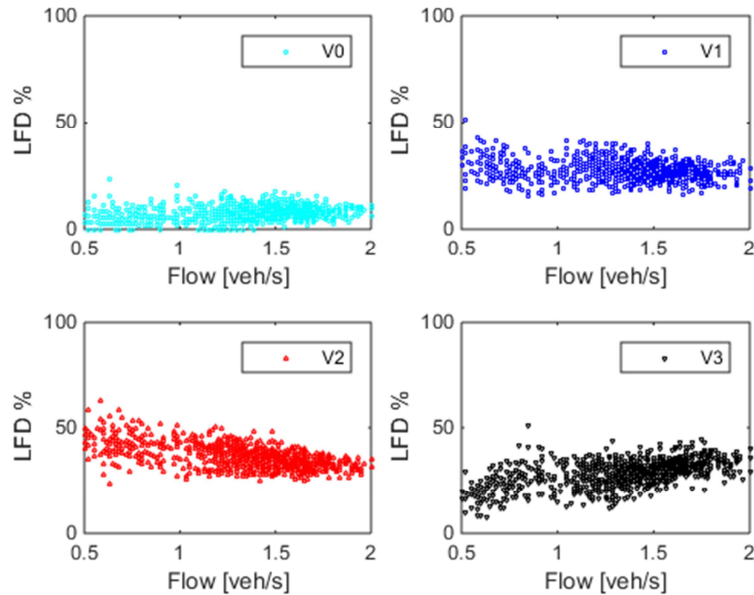


Figure 7: Percentage of lane flow distribution (LFD %) in function of average total flow [veh/s] on 14/11 at detector no.1.

Furthermore, when traffic flow increase (2 [veh/s]), the lane V0 carries about 10% of traffic (coming from the on ramp), 25% of vehicles circulate on lane V1, 28% of vehicles circulate on lane V2 and 37% of vehicles circulate on lane V3.

The same percentage of flow per lane, for V1, V2 and V3, could be noticed even when the shoulder is closed. Generally, it can be distinguished that the percentage of LFD of lanes V1 and V2 decreases when the average flow increases. In contrast, the LFD% of lane V3 increases when the average flow increases.

VIII. LANE FLOW DISTRIBUTION DURING THE DAY

Figure 8 shows the percentage of lane flow distribution in term of hours during the day from 5 AM to 10 PM

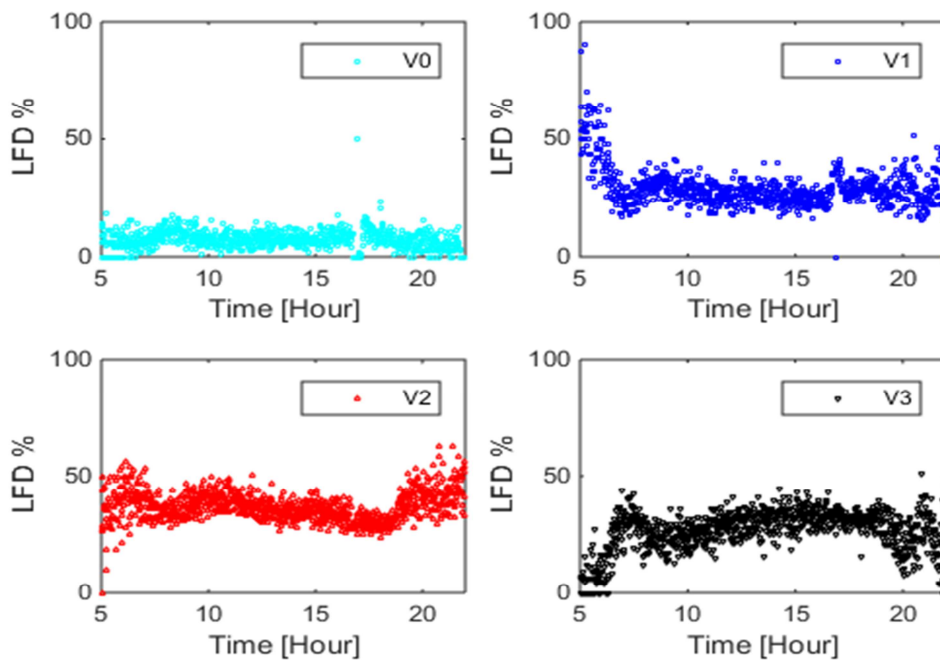


Figure 8: Percentage of lane flow distribution (LFD %) in function of time [Hours] from 5 AM to 10 PM on 14/11 at detector no.1.

PM. It is noticeable that the percentage of the lane flow distribution varies during the day depending on the situation of traffic flow. When traffic flow is congested, the percentage of LFD of lanes V2 and V3 decreases while the percentage of LFD of V0 and V1 increases. In addition, when traffic flow is around its maximum value, the percentages of the LFD of V1, V2 and V3 are more or less equivalent.

IX. EFFECT OF VSLs ON THE LFD WHEN THE SHOULDER IS OPEN OR CLOSED

The VSL means imposing a speed lower than the maximum speed limit in order to create a better traffic condition. It could be useful in two different cases:

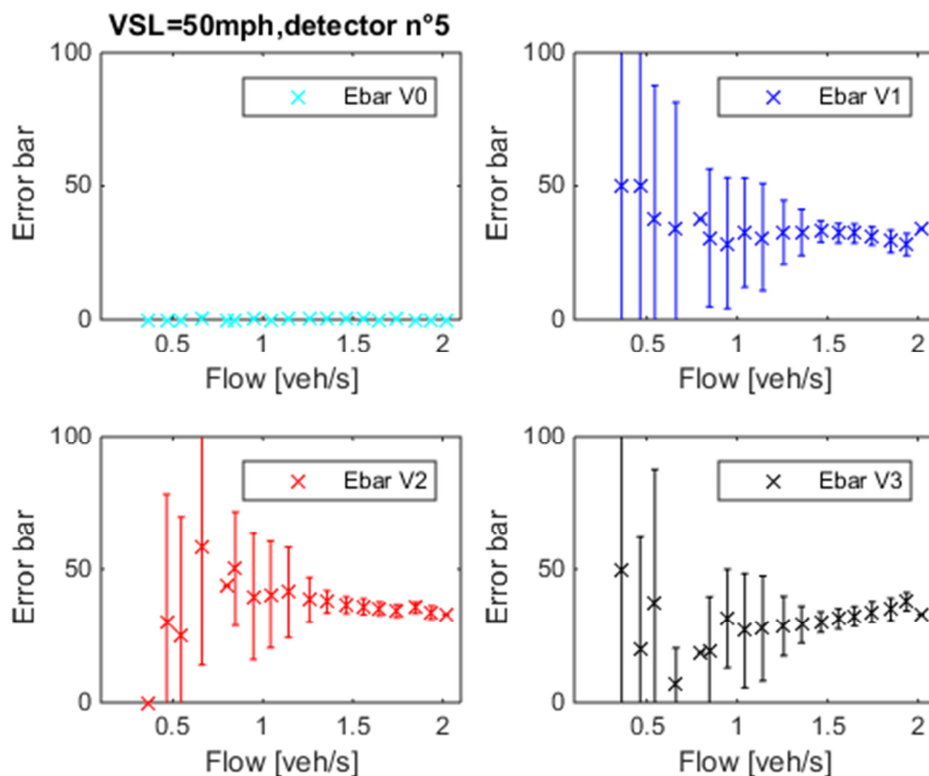
- When the traffic situation is fluid, the aim is to delay the appearance of congestion.
- When the traffic is congested, the aim is to advance the disappearance of congestion.

However, all data have been aggregated during the working day when the weather is ideal (i.e. the road is dry). In that case, having this amount of data (see **Figure 6**) would be sufficient in order to analyze the effect of VSLs when the shoulder is open or closed:

I. DYNAMIC LANE (SHOULDER LANE) IS CLOSED

Figure 9 shows the data collected from detector no. 5, the following notes can be noted:

- The VSL is activated when the average traffic flow is increasing. When traffic flow is weak (0-0.5 [veh/s]), for the VSLs (50 mph and 60 mph), there is not enough data sample because there is no need to reduce the speed limit (see **Figure 9**). The road structure can cope with the demand. It means that the ATM (Active Traffic Management) system works and well implemented.
- The percentage of LFDs are compared with different speed limit (70 mph maximum speed limit, 60 and 50 mph) based on available data. Sufficient data are presented in the following intervals (see **Figure 9**):
 - (1.1-1.7 [veh/s]) to compare the LFD between VSL (50 mph) and VSL (60 mph).
 - (0.9-1.5 [veh/s]) to compare the LFD between VSL (60 mph) and VSL (70 mph (when speed is 70 mph, it means that the VSL is not activated)).
 - (1.1-1.5 [veh/s]) to compare the LFD between VSL (50 mph) and VSL (70 mph).



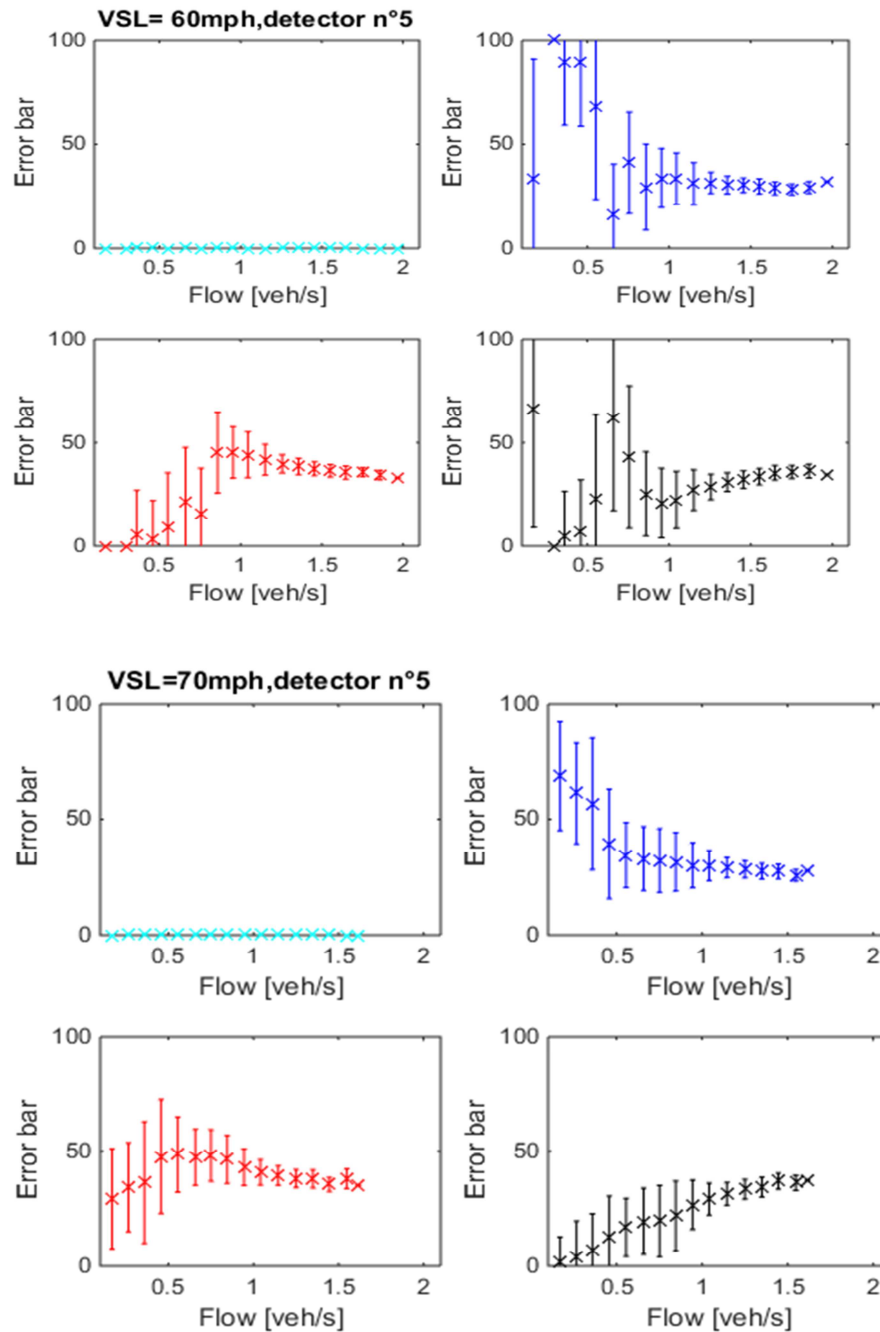


Figure 9: Standard deviation of LFD per lane in function of average flow at detector no. 5 with VSL 50/60 mph and without VSL 70 mph. Note: Error bar shows the mean value of LFD% (cross (x) points) and the upper and lower limit of standard deviation of LFD% at each value of flow. At a particular flow, the summation of the mean value of LFD% of all the lanes are equal to 100%, for instance, for VSL 50 mph, at flow = 1 veh/s, the LFD% of; V0 = 0; V1= 30mph; V2=45mph; V3=25mph. Then the sum is 0+30+45+25=100%.

COMPARISON OF LFD% BETWEEN VSLs 50/60 MPH AND VSL 70 MPH

In (Figures 10 & 11), the difference of LFD% between the VSL 50/60 mph and 70 mph at detector No. 6 is demonstrated. The comparisons have been done using the following equations:

$$\text{Difference of \%LFD (50/60)} = \%LFD \text{ of } V_i \text{ (VSL 60 mph)} - \%LFD \text{ of } V_i \text{ (VSL 50 mph)}$$

$$\text{Difference of \%LFD (50/70)} = \%LFD \text{ of } V_i \text{ (VSL 70 mph)} - \%LFD \text{ of } V_i \text{ (VSL 50 mph)}$$

$$\text{Difference of \%LFD (60/70)} = \%LFD \text{ of } V_i \text{ (VSL 70 mph)} - \%LFD \text{ of } V_i \text{ (VSL 60 mph)}$$

Where i = number of lanes, 0, 1, 2, and 3.

Using these equations, it is shown that when traffic flow increases:

- The lane V1 is less used with VSL 70 mph (maximum safe speed).
- The lane V2 is more used with VSLs (50/60 mph).

- The lane V3 is less used with VSL 50 mph. used especially when the traffic flow increases. It is deducible that when VSL is activated, the difference of LFD% becomes smaller. In another word, the LFD% would be more homogeny. This improvement helps the infrastructure to carry larger number of vehicle and reduce congestion.

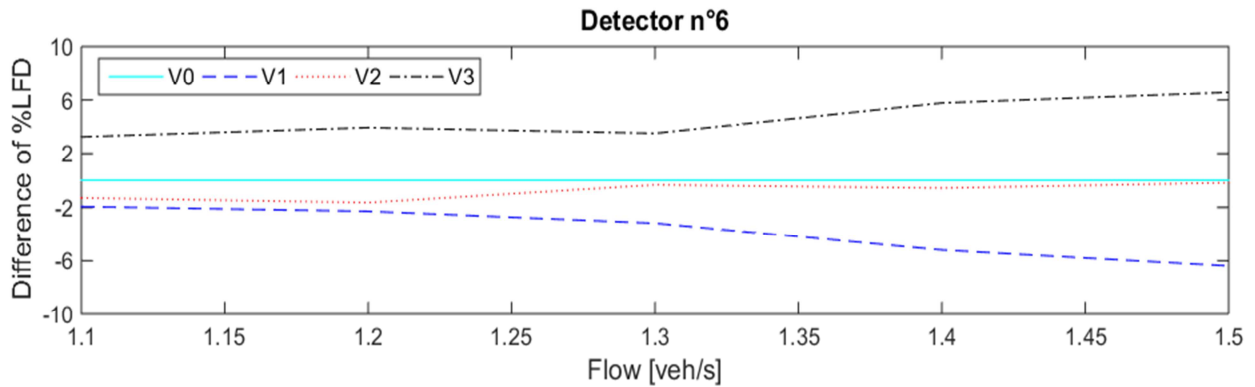


Figure 10: The difference of the %LFD at all the circulating lanes, when the VSL is 70 mph and 50 mph at detector No. 6.

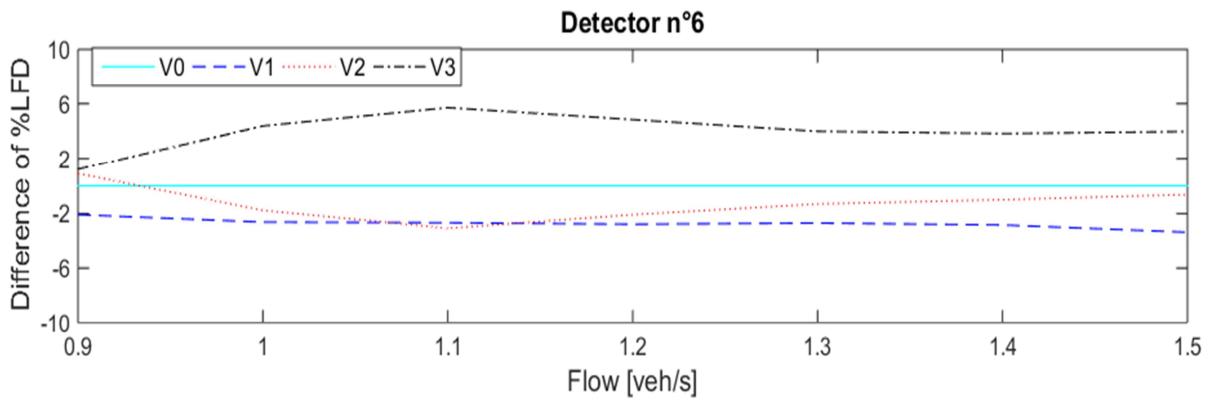


Figure 11: The difference of the %LFD at all the circulating lanes, when the VSL is 70 mph and 60 mph at detector No. 6.

COMPARISON OF LFD% BETWEEN VSL 50 MPH AND VSL 60 MPH

In (Figure 12), the difference of LFD% between the VSL 50 mph and 60 mph at detector No. 6 is shown. It is comprehensible that when traffic flow increases:

- The lane V1 is less utilized while the lane V3 is more utilized with VSL 60 mph. It means that changing the VSL value from 50 mph to 60 mph affects the LFD% of slow lane V1 and median lane V3 considerably.
- The lane V2 is more utilized when VSL is 60 mph.

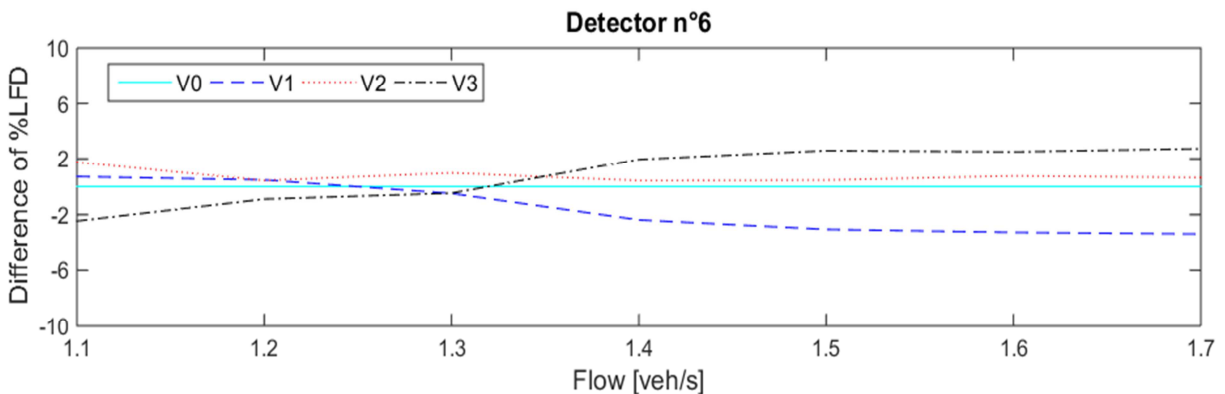


Figure 12: The difference of the %LFD at all the circulating lanes, when the VSL is 50 mph and 60 mph at detector No. 6.

It can be concluded that, the VSLs can change the LFD% of the lanes. It increases the LFD% of lane V1 which is under-utilized when the traffic flow is low. It decreases the LFD% of lane V3 which is overloaded when traffic flow is high. It means that the VSLs can homogenize the traffic flow over all the circulating lanes with any amount of demand.

2. DYNAMIC LANE (SHOULDER LANE) IS OPEN

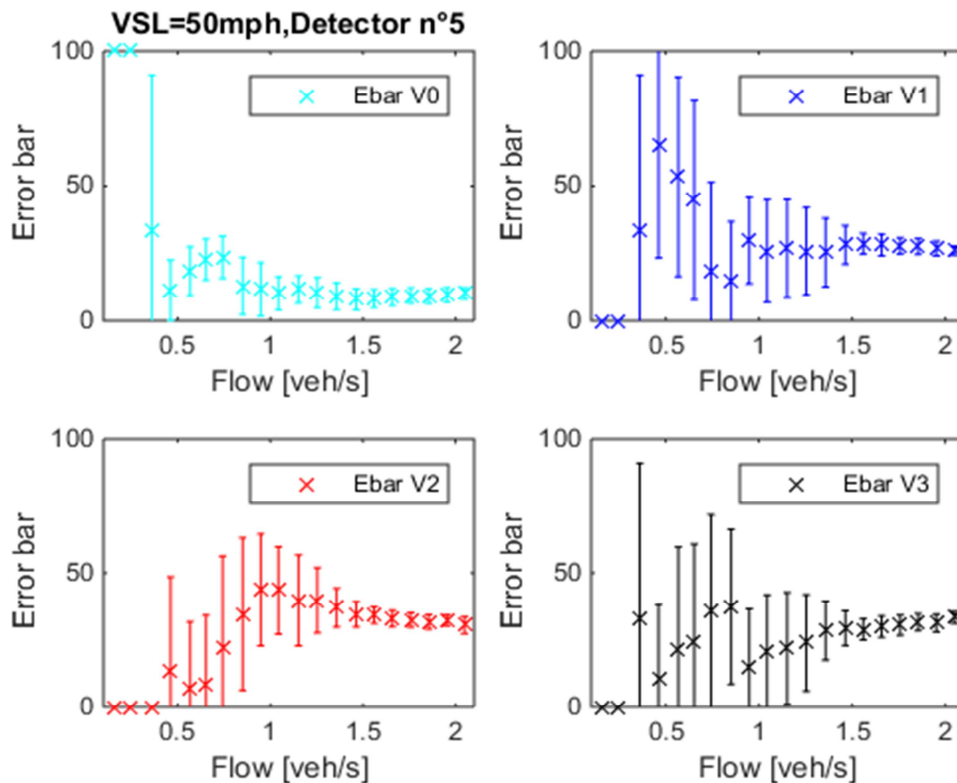
In this section all the data analyzed in case where the dynamic lane is open. The LFD% in function of traffic flow [veh/s] for the maximum speed 70 mph and the VSL 50/60 mph at detector No. 5 is demonstrated in (Figure 13), the following facts can be identified:

- The data samples are not satisfactory in the case of maximum speed limit (70 mph). It is normal, because the shoulder lane is open to traffic and the VSLs are activated when traffic demand is high. Therefore, there is no need that the shoulder lane be open when the road could be managed with maximum speed limit (70 mph).
- The LFD% of the lane V0 is about 10% when the VSL is activated. It is perceptible that in case when dynamic lane is open the road infrastructure can handle more traffic demand and can alleviate or eliminate congestion during high traffic demand.

COMPARISON OF LFD% BETWEEN VSL 50/60 MPH AND 70 MPH

Figure 14 shows the difference of the LFD% in function of traffic flow between the VSL 50 mph and the VSL 60 mph at detector No. 6. When traffic flow increases, the following facts are observable:

- The lane V0 is less utilized, but the V3 is used more with VSL 60 mph.
- The lane V1 is operated less while the lane V2 is more utilized with VSL 60 mph. It means that, changing the VSL value from 50 mph to 60 mph or vice-versa affects considerably the lanes V1 and V2.



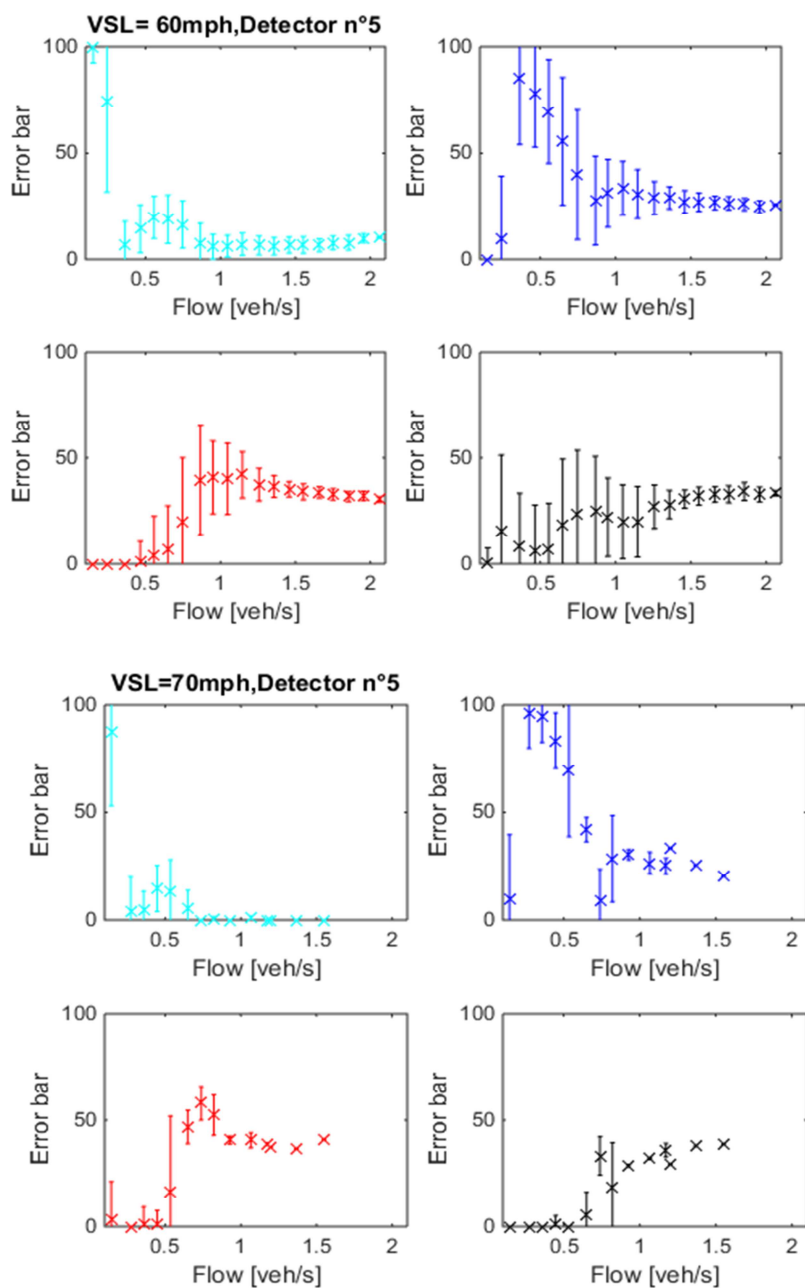


Figure 13: Standard deviation of LFD% per lane in function of average flow [veh/s] at detector no. 5 with VSL 50/60 mph and without VSL 70 mph. Note: Error bar shows the mean value of LFD% (cross (x) points) and the upper and lower limit of standard deviation of LFD% at each value of flow. At a particular flow, the summation of the mean value of LFD% of all the lanes are equal to 100%, for instance, for VSL 50 mph, at flow = 1 veh/s, the LFD% of; V0 = 10; V1= 30mph; V2=48mph; V3=12mph. Then the sum is 10+30+48+12=100%.

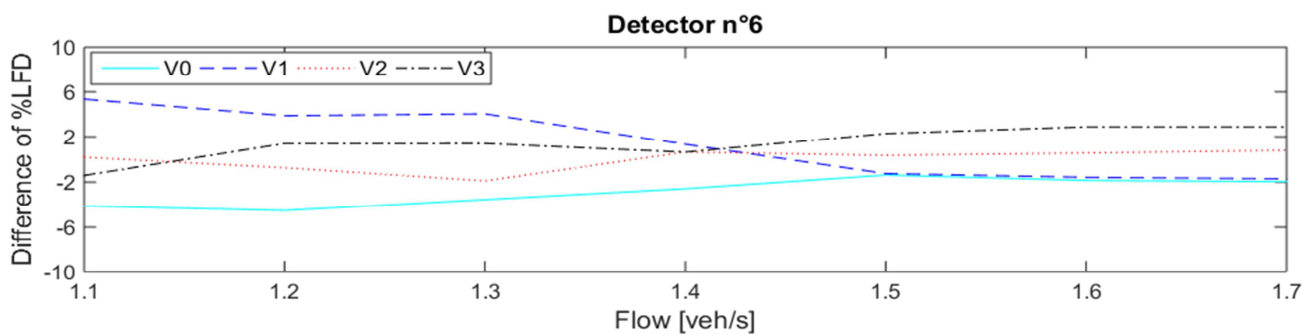


Figure 14: Difference of the LFD% at all the circulating lanes when the VSL are 50 mph and 60 mph at detector No. 6 (the shoulder is open).

THE IMPACT OF SHOULDER ON THE LFD% WITH DIFFERENT VSL

In **Figure 15**, the difference of LFD% is illustrated when the shoulder is open or closed at detector No. 10. When the dynamic lane is open, it can be observed that:

- The lanes V1, V2 and V3 are less utilized when a VSL is activated (50/60 mph). Therefore, the dynamic lane can reduce traffic flow at all carriageway lanes.
- When the maximum speed limit (70 mph) is applied: the data samples are not sufficient to conclude the affect of maximum speed limit on the utilization of circulating lanes. As mentioned before; it is normal that the shoulder lane is open to traffic and the VSLs are activated when traffic demand is high. Therefore, there is no need to the shoulder lane to be open when the road could be managed with maximum speed limit (70 mph).
- The shoulder can carry about 10% of traffic flow.
- With VSL 50 mph; the percentage of LFD of lane V0 is greater comparing with VSL 60 mph.

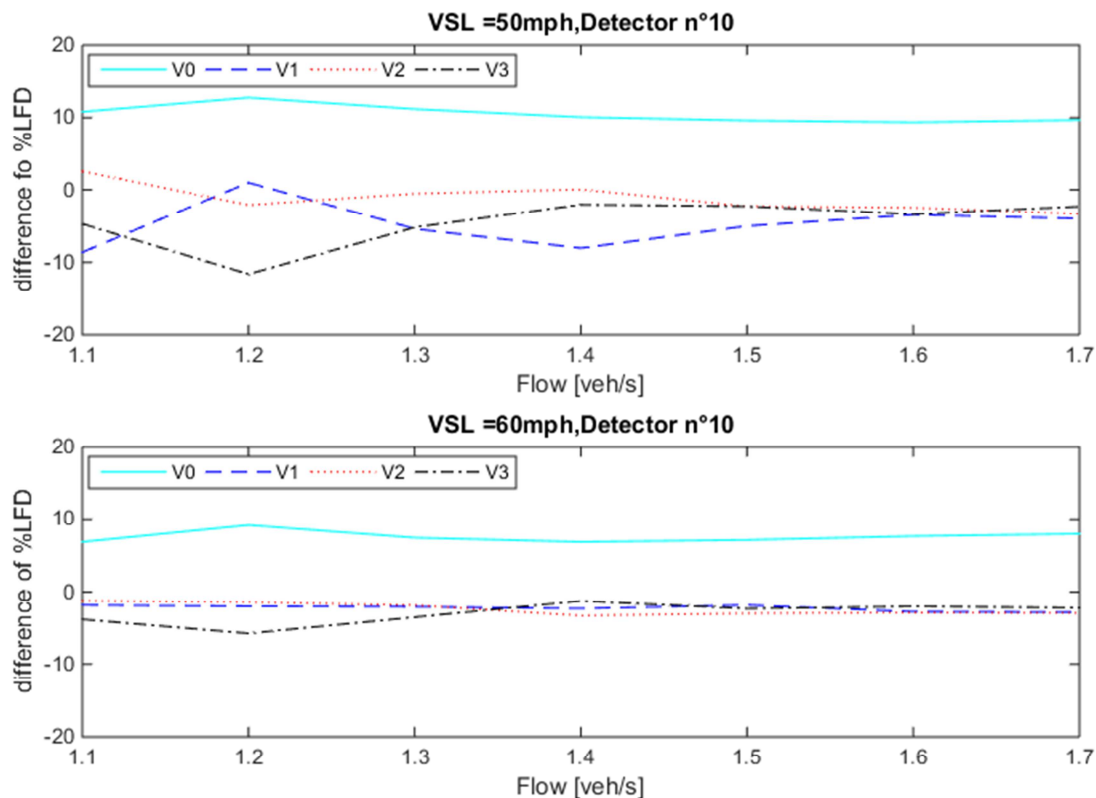


Figure 15: The difference of the %LFD at all the circulating lanes, when the VSL is 50 mph and 60 mph at detector No. 10.

X. CONCLUSION

The M42 motorway in UK on which the data was collected is a modern highway with real-time traffic management system "Active Traffic Management" therefore it could be counted as a reliable study area. In this study, some new results concerning the LFD were obtained. Sufficient amount of data was available and accurate collected using loop detectors that have allowed presenting some of the following results. Firstly, the characteristics of LFD through a section of motorway can be discovered easily. Secondly, the effect of VSL is studied on changing the LFD% at carriageway lanes. Lastly, the impact of opening and closing of shoulder is studied. Results showed that dynamic lane can improve traffic flow and stay in free flow zone. For instance, congestion could be avoided. The VSLs have, also, a positive effect on the homogenization of traffic flow among the carriageway lanes. The impact of these two measurements together (dynamic lane and VSLs) could be more efficient in order to traffic flow stay in free flow zone.

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